浙江大學 海洋学院
OCEAN COLLEGE, ZHEJIANG UNIVERSITY

**2024 SPHERIC Zhuhai International Workshop**

KYOTO UNIVERSITY

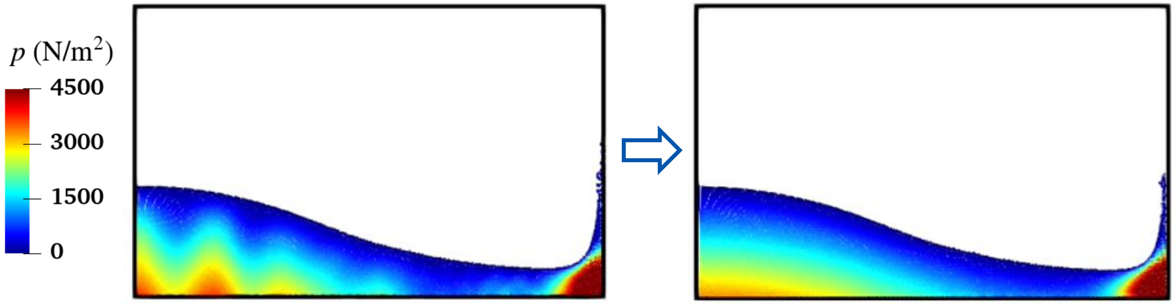
# An enhanced DualSPHysics with improvements in accuracy, energy conservation and resolution of the continuity equation

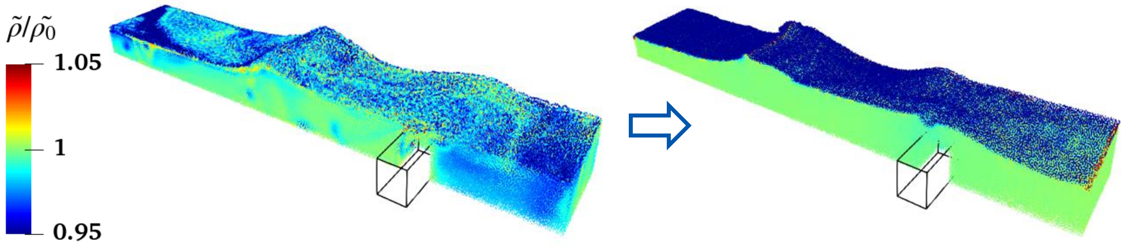

Yi Zhan[1], Min Luo[1], Abbas Khayyer[2]

[1] Ocean College, Zhejiang University

[2] Department of Civil and Earth Resources Engineering, Kyoto University


18th October 2024



# 1 Background

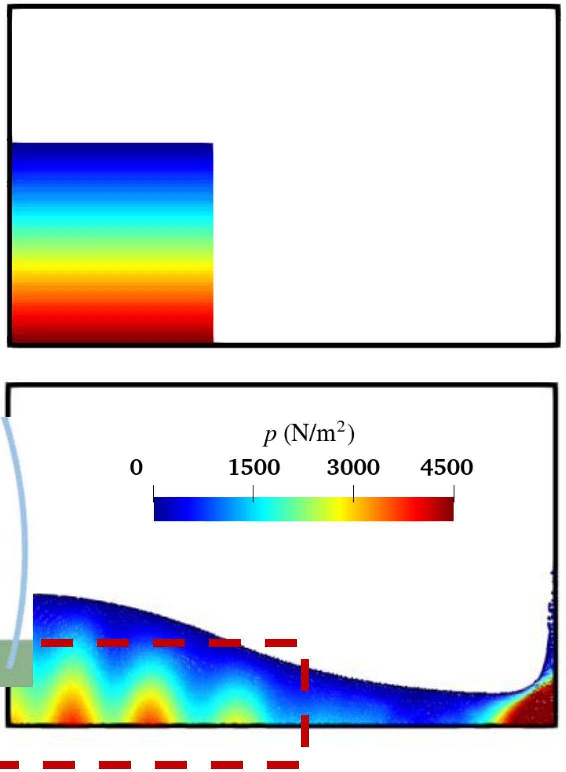
➢ Dam-break flow

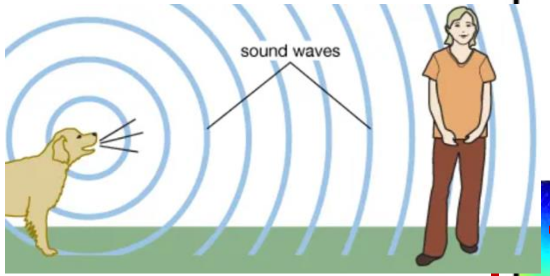
WCSPH

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \boldsymbol{u}$$

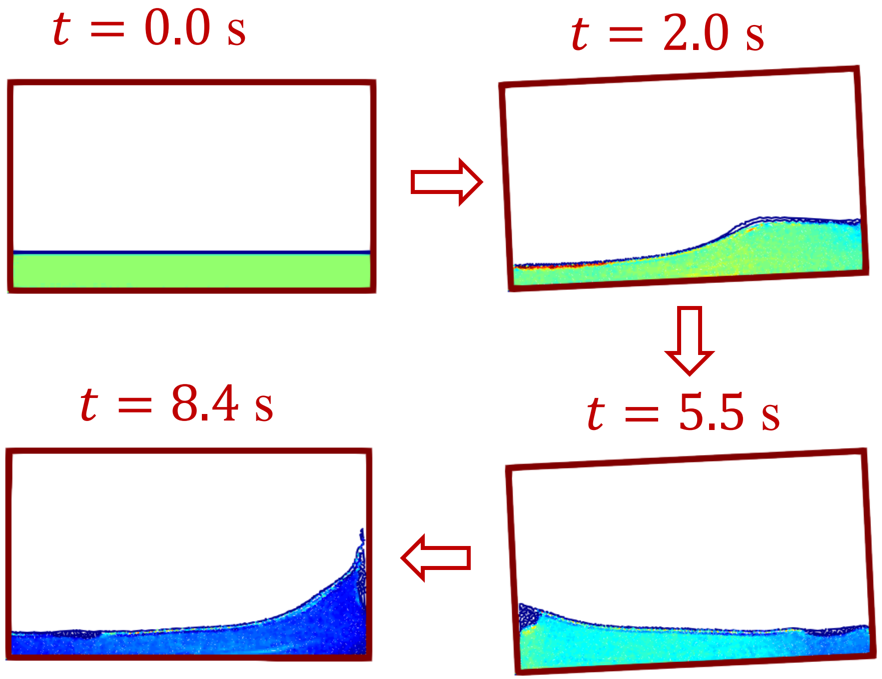
$$\frac{D\boldsymbol{u}}{Dt} = -\frac{\nabla p}{\rho} + \boldsymbol{g}$$

$$p = c_0^2(\rho - \rho_0)$$

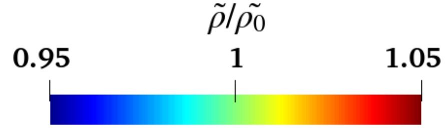



➢ Sloshing flow




$$\tilde{\rho}_a = \sum_b m_b W_{ab}$$




■ Oscillations of pressure (acoustic wave + numerical discretization errors)

■ Volume non-conservation during simulation

# 2 Developments of DualSPHysics+: Overview

## $\delta$-SPH model in DualSPHysics

$$\frac{D\rho_a}{Dt} = -\rho_a \sum_b \boldsymbol{u}_{ab}\cdot\nabla_a W_{ab} V_b + D_a,\quad \frac{D\boldsymbol{u}_a}{Dt} = -\sum_b m_b\left(\frac{p_b+p_a}{\rho_b\rho_a}+\Pi_{ab}\right)\nabla_a W_{ab}+\boldsymbol{g}$$

DualSPHysics

Further enhancements corresponding to four aspects

i) Minimum energy dissipation ← Riemann stabilization term instead of artificial viscosity

ii) Consistent particle shifting at and in the vicinity of free surface ← OPS

iii) Enhanced resolution of the continuity equation ← VEM and VCS

iv) Effective cleaning of velocity-divergence errors ← Combination of VEM and HPDC

## $\delta$R-SPH-OPS-VCS-VEM-HPDC in DualSPHysics+ [1]

$$\frac{D\rho_a}{Dt} = -\rho_a\sum_b \boldsymbol{u}_{ab}\cdot\nabla_a W_{ab}V_b + D_a,\quad \frac{D\boldsymbol{u}_a}{Dt} = -2\sum_b m_b\left(\frac{p^*}{\rho_a\rho_b}\right)\nabla_a W_{ab}+\boldsymbol{g}+\boldsymbol{a}_a^{\mathrm{VEM}}-\nabla\psi_a,\quad p^*=\frac{1}{2}F(p_L,p_R)+\frac{1}{2}\phi\bar{\rho}(u_L-u_R),\quad \boldsymbol{r}_a'=\boldsymbol{r}_a+\delta\boldsymbol{r}_a^{\mathrm{OPS}}+\delta\boldsymbol{r}_a^{\mathrm{VCS}}$$

[1] Zhan, Yi, Min Luo, and Abbas Khayyer. "DualSPHysics+: An enhanced DualSPHysics with improvements in accuracy, energy conservation and resolution of the continuity equation." *Computer Physics Communications* (2024): 109389.

# 2.1 DualSPHysics+: Combination of VEM and HPDC

## ➤ Velocity-divergence Error Mitigating (VEM) [1]

$$\boxed{p_a^{\mathrm{VEM}}} = c_s^2 d\rho_a = c_s^2 \Delta t \left(\frac{D\rho}{Dt}\right)_a^{k-1} = -\rho_a c_s^2 \Delta t \boxed{\langle \nabla \cdot \boldsymbol{u} \rangle_a^{k-1}}$$

Pressure related to velocity divergence error

This error will be accumulated

$$\boldsymbol{a}_a^{\mathrm{VEM}} = -\frac{1}{\rho_a}\sum_b F\left(p_a^{\mathrm{VEM}}, p_b^{\mathrm{VEM}}\right)\nabla_a W_{ab} V_b$$

$$F(p_a, p_b) = \begin{cases} p_b + p_a & (p_a \geq 0 \cup a \notin \Omega_{IN}) \\ p_b - p_a & (p_a < 0 \cap a \in \Omega_{IN}) \end{cases}$$

$$\frac{D\boldsymbol{u}_a}{Dt} = -\sum_b m_b \left(\frac{F(p_a, p_b)}{\rho_a \rho_b} + \Pi_{ab}\right)\nabla_a W_{ab} + \boldsymbol{g}_a + \boxed{\boldsymbol{a}_a^{\mathrm{VEM}}}$$

## ➤ Hyperbolic/Parabolic Divergence Cleaning (HPDC) [2]

$$\begin{cases} \left(\dfrac{D\boldsymbol{u}}{Dt}\right)_\psi + \nabla \psi = 0 \\ \mathcal{D}(\psi) + \nabla \cdot \boldsymbol{u} = 0 \end{cases}$$

Hyperbolic term

Parabolic term

$$\frac{\partial^2 (\nabla \cdot \boldsymbol{u})}{\partial t} - \boxed{c_h^2 \nabla^2 (\nabla \cdot \boldsymbol{u})} + \boxed{\frac{c_h^2}{c_p^2}\frac{\partial (\nabla \cdot \boldsymbol{u})}{\partial t}} = 0$$

$$\frac{D\boldsymbol{u}_a}{Dt} = -2\sum_b m_b \left(\frac{p^*}{\rho_a \rho_b}\right)\nabla_a W_{ab} + \boldsymbol{g} + \boldsymbol{a}_a^{\mathrm{VEM}} \boxed{-\nabla \psi_a}$$

$\psi = 0$ for boundary particles


[1] Khayyer et al. "Enhanced resolution of the continuity equation in explicit weakly compressible SPH simulations of incompressible free-surface fluid flows." *Applied Mathematical Modelling* 116 (2023): 84-121.

[2] Fourtakas et al. " An investigation on the divergence cleaning in weakly compressible SPH. " Proceedings of the 17th International SPHERIC

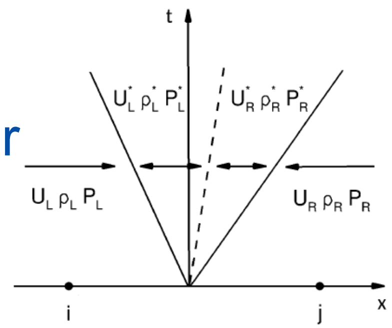
# 2.2 DualSPHysics+: $\delta$R-SPH

Artificial viscosity

$$\frac{D\boldsymbol{u}_a}{Dt} = -\sum_b m_b\left(\frac{p_b+p_a}{\rho_b\rho_a} + \boxed{\Pi_{ab}}\right)\nabla_a W_{ab} + \boldsymbol{g}$$

$\Downarrow$ Riemann solver

$$\frac{D\boldsymbol{u}_a}{Dt} = -2\sum_b m_b\left(\boxed{\frac{p^*}{\rho_a\rho_b}}\right)\nabla_a W_{ab} + \boldsymbol{g}$$




Implicitly and properly introduce minimum dissipation via Riemannian solvers

- Traditional Riemann solver for continuity equation

$$\frac{D\rho_i}{Dt} = -\rho_i\sum_j \boldsymbol{u}_{ij}\cdot\nabla_i w_{ij}V_j + D_i^R, \ \boxed{D_i^R} = 2\rho_i c_0\sum_j \frac{\rho_j-\rho_i}{\rho_j+\rho_i}\frac{\boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|}\cdot\nabla_i w_{ij}V_j$$

$$\sum_i D_i^R V_i = \sum_i\sum_j 2\rho_i c_0 \frac{\rho_j-\rho_i}{\rho_j+\rho_i}\frac{\boldsymbol{r}_{ij}}{|\boldsymbol{r}_{ij}|}\cdot\nabla_i w_{ij}V_j V_i \boxed{\neq 0}$$

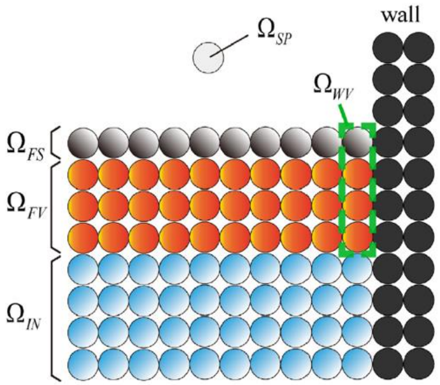
Diffusive term is not globally conserved [1]

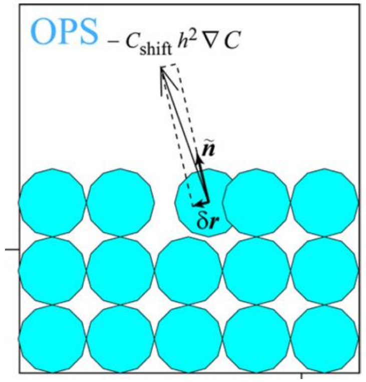
- The combination of the $\delta$-SPH density diffusion term in the continuity equation and a Riemann stabilization term in the momentum equation is adopted


[1] Khayyer et al. "An improved Riemann SPH-Hamiltonian SPH coupled solver for hydroelastic fluid-structure interactions." *Engineering Analysis with Boundary Elements* 158 (2024): 332-355.

# 2.3 DualSPHysics+: Particle shifting

## ➢ Optimized Particle shifting (OPS) [1]




I. Conduct particle type detections

II. Compute shifting vector based on Fick's law of diffusion

III. Eliminate the normal shifting component for $\Omega_{FS}, \Omega_{FV}$

$$\boxed{C_a} = \sum_b W_{ab} V_b, \quad \delta \boldsymbol{r}_a^{\text{OPS}} = -C_{\text{shift}} h^2 \left( \boldsymbol{I} - \tilde{\boldsymbol{n}}_a \otimes \tilde{\boldsymbol{n}}_a \right) \cdot \nabla C_a$$

Particle concentration

## ➢ Volume Conservation Shifting (VCS) [2]

$$\tilde{\rho}_a = \sum_b m_b W_{ab}$$

Volume conservation error ⇩

$$\left\{ \nabla \cdot \left( \frac{\nabla p^{\text{VCS}}}{\rho} \right) \right\}_a^{k+1} = \boxed{\frac{\tilde{\rho}_0 - \tilde{\rho}_a^k}{\tilde{\rho}_0 \Delta t^2}} + \frac{1}{\Delta t^2 \tilde{\rho}_0 c_s^2} p_a^{\text{VCS},k+1}$$

Explicitly solved ⇩

$$\delta \boldsymbol{r}_a^{\text{VCS}} = \Delta t^2 \left( -\frac{1}{\rho_a} \left\langle \nabla p^{\text{VCS}} \right\rangle_a^{\text{L}} \right)$$

$$\boldsymbol{r}_a' = \boldsymbol{r}_a + \boxed{\delta \boldsymbol{r}_a^{\text{OPS}}} + \boxed{\delta \boldsymbol{r}_a^{\text{VCS}}}$$


[1] Khayyer et al. "Comparative study on accuracy and conservation properties of two particle regularization schemes and proposal of an optimized particle shifting scheme in ISPH context." *Journal of Computational Physics* 332 (2017): 236-256.

[2] Khayyer et al. "Enhanced resolution of the continuity equation in explicit weakly compressible SPH simulations of incompressible free-surface fluid flows." *Applied Mathematical Modelling* 116 (2023): 84-121.

# 3 Numerical validations and applications

## ➢ Four cases

[1] A rotating fluid square

[2] Impact of two fluid patches

[3] Two-dimensional dam break

[4] Three-dimensional dam break

## ➢ References

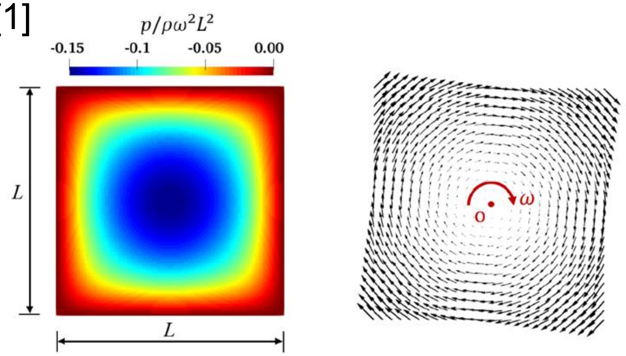

[1] Le Touzé, D., et al. "A critical investigation of smoothed particle hydrodynamics applied to problems with free-surfaces." *IJNMF* (2013)

[2] Antuono et al. "Energy balance in the δ-SPH scheme." *CMAME* (2015)

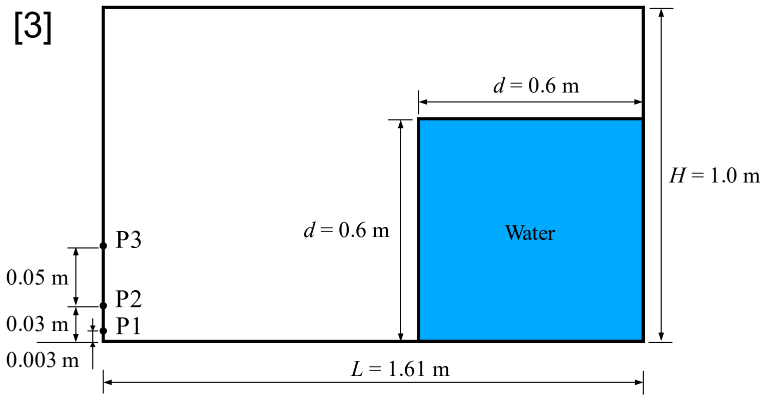
[3] Lobovský, Libor, et al. "Experimental investigation of dynamic pressure loads during dam break." *JFS* (2014)

[4] Kleefsman, K. M. T., et al. "A volume-of-fluid based simulation method for wave impact problems.“ *JCP* (2005)

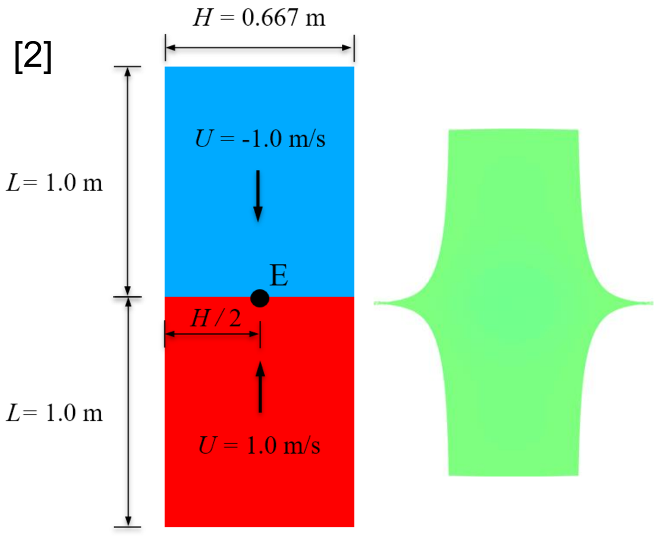

[1]



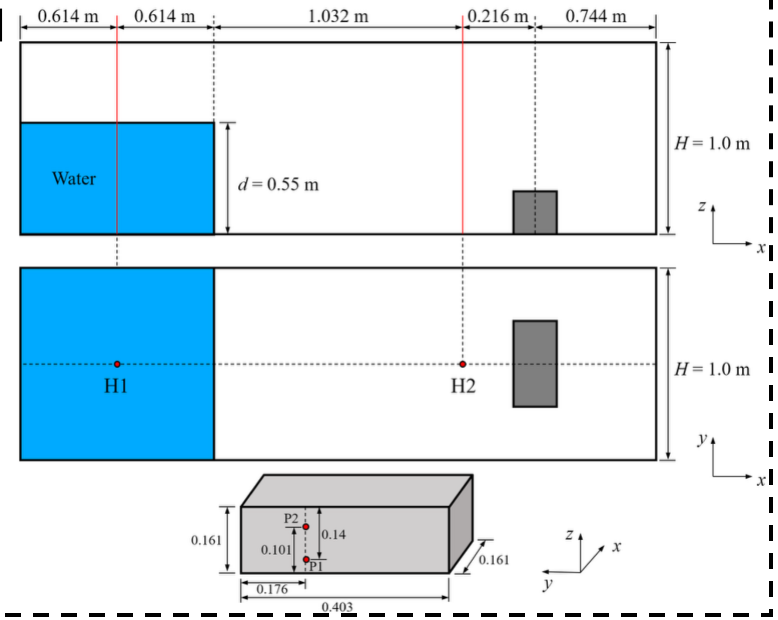

[3]



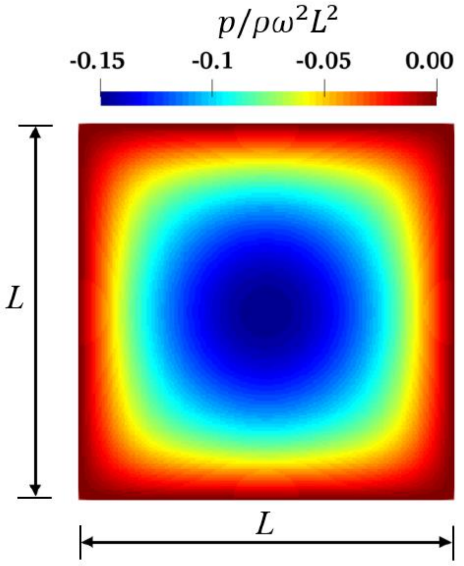

[2]

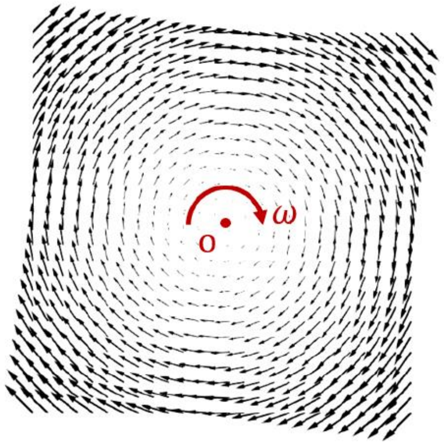



[4]



# 3.1 A rotating fluid square: Numerical settings




$L$ = 1 m, $dp$ = 0.02, 0.01, 0.005 m

$\omega$ = 1 rad/s, $C_0$ = 15.0 m/s

Predictor-Corrector, $C_{CFL}$ = 0.3

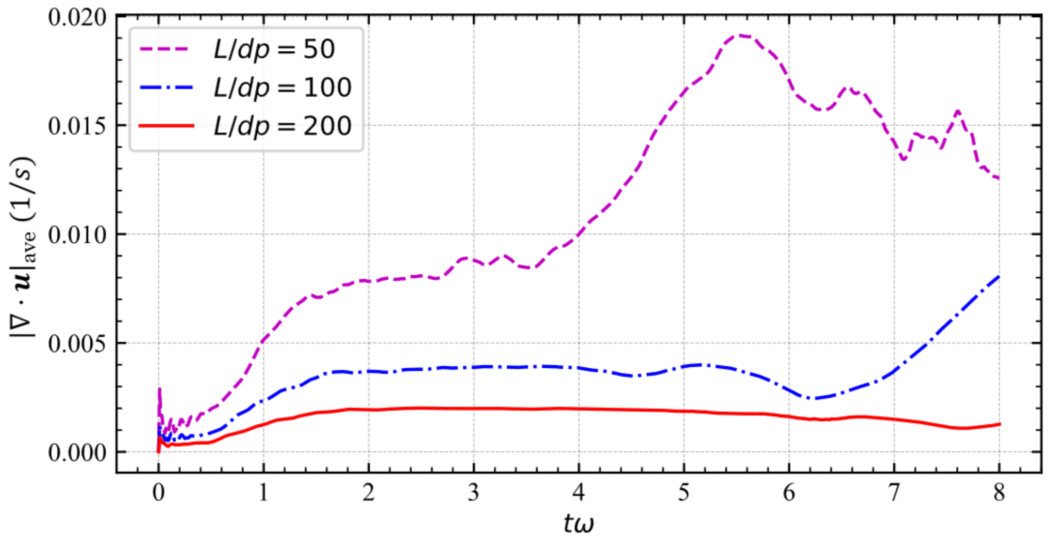
Kernel = 5th Wendland C2, $h/dp$ = 2

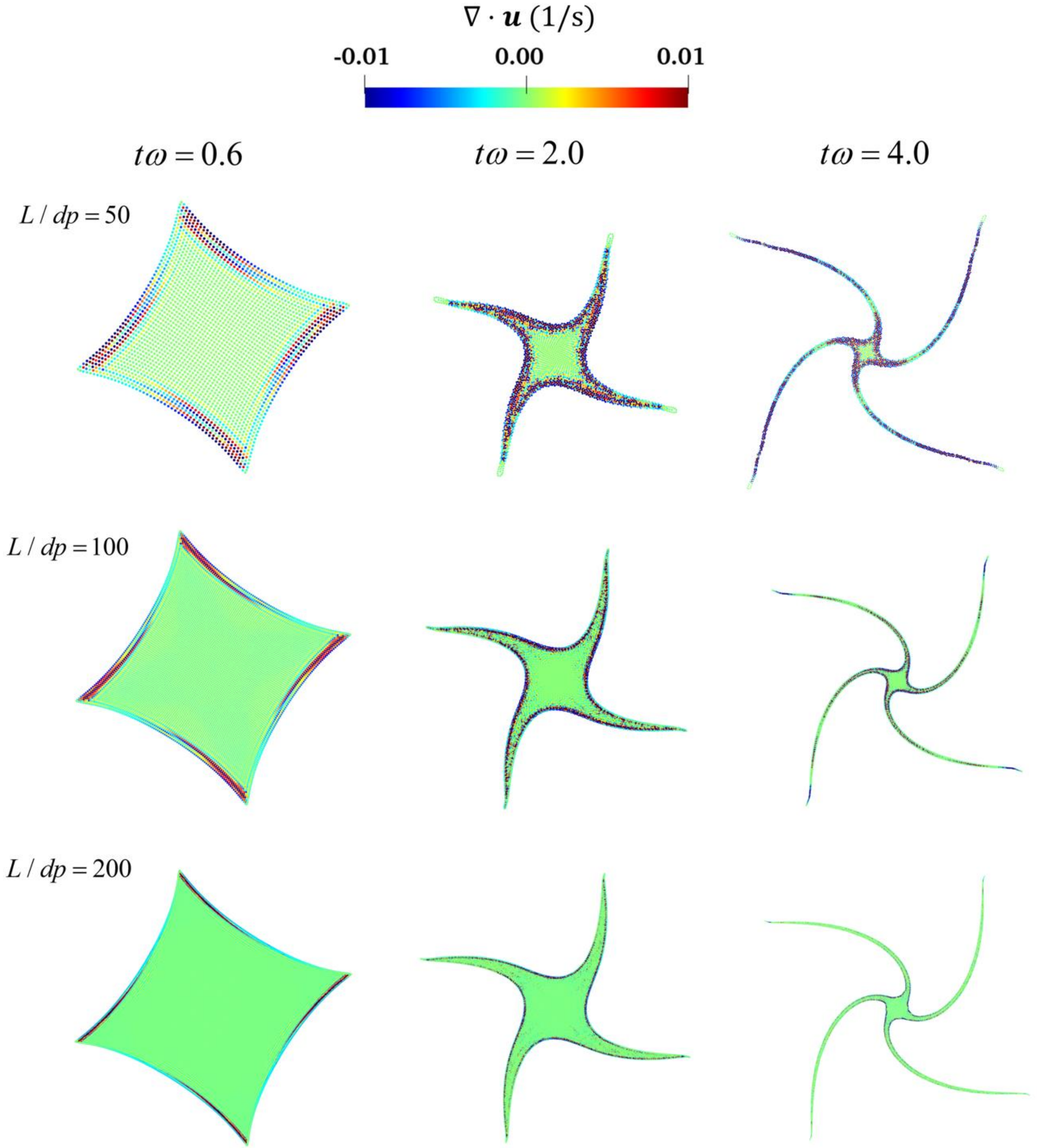
$$\boldsymbol{u}(x, y, 0) = (\omega y; -\omega x)$$

$$p_0(x,y) = \rho \sum_m^\infty \sum_n^\infty \frac{-32\varpi^2}{mn\pi^2\left[(n\pi/L)^2 + (m\pi/L)^2\right]} \sin\left(\frac{m\pi x^*}{L}\right) \sin\left(\frac{n\pi y^*}{L}\right) \quad m, n \in \mathrm{N}_{odd}$$

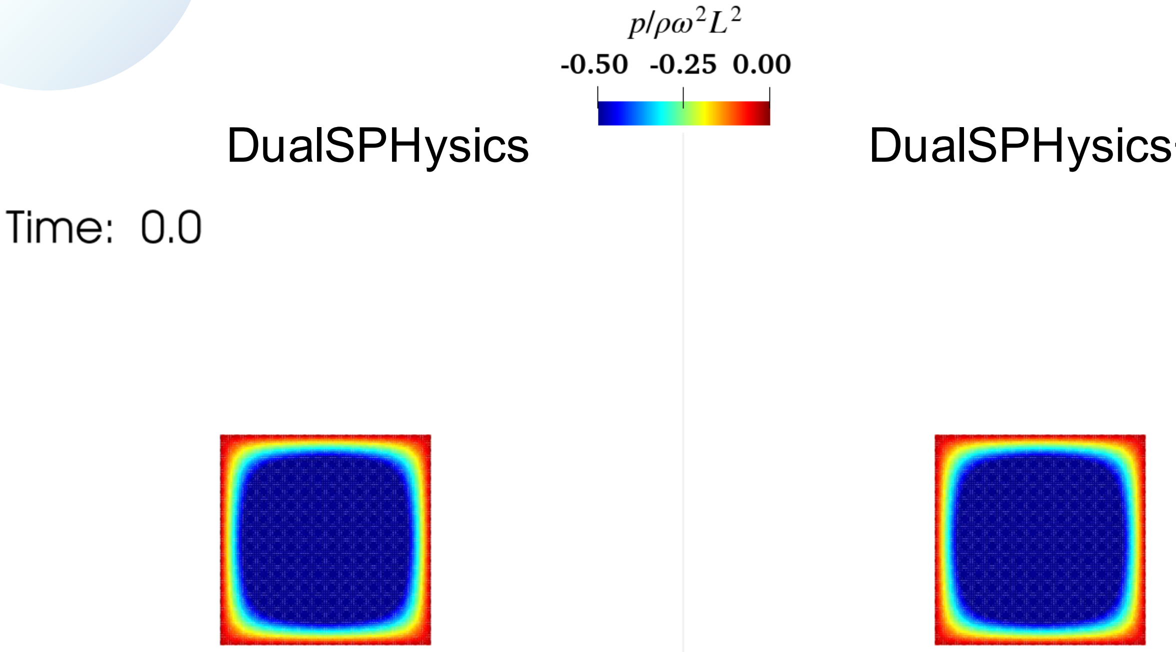
# 3.1 A rotating fluid square: Convergence analysis

- Pressure field smoother and highly-deformed tip shapes closer to the reference results
- Effectiveness and convergence of velocity divergence cleaning

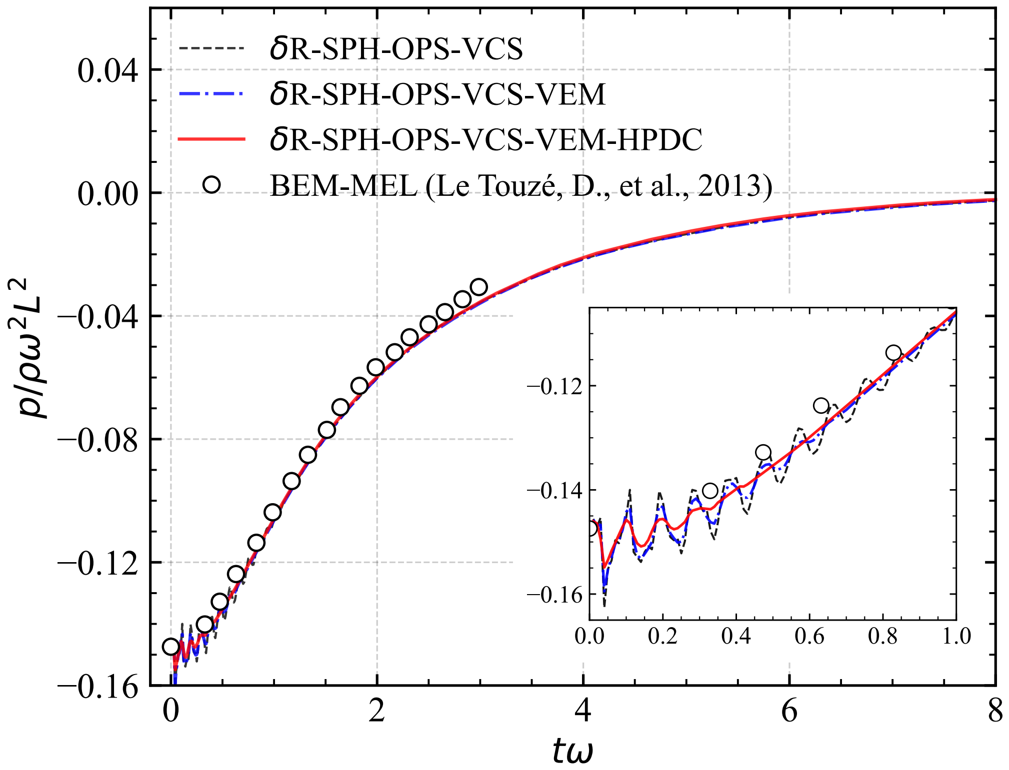


# 3.1 A rotating fluid square: Pressure



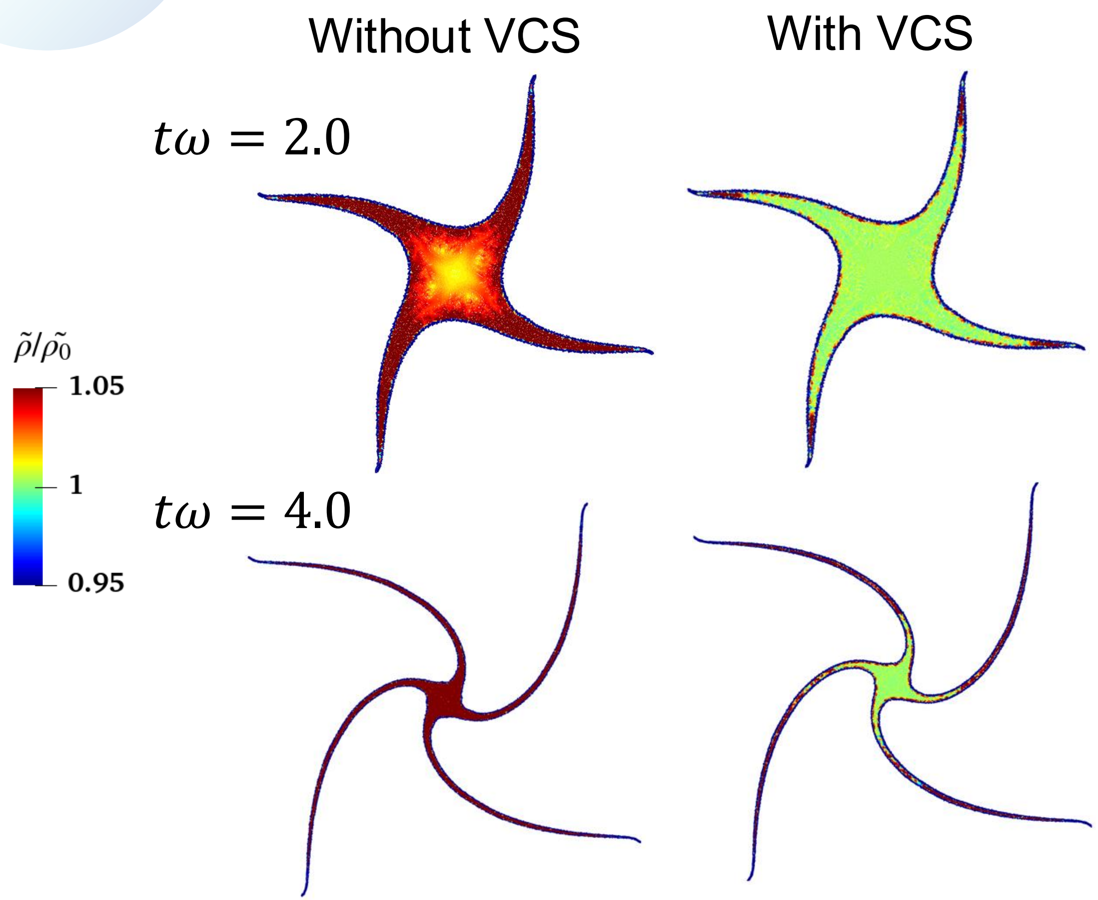

➢ Pressure histories at the square center




- DualSPHysics+ improves the numerical stability and accuracy

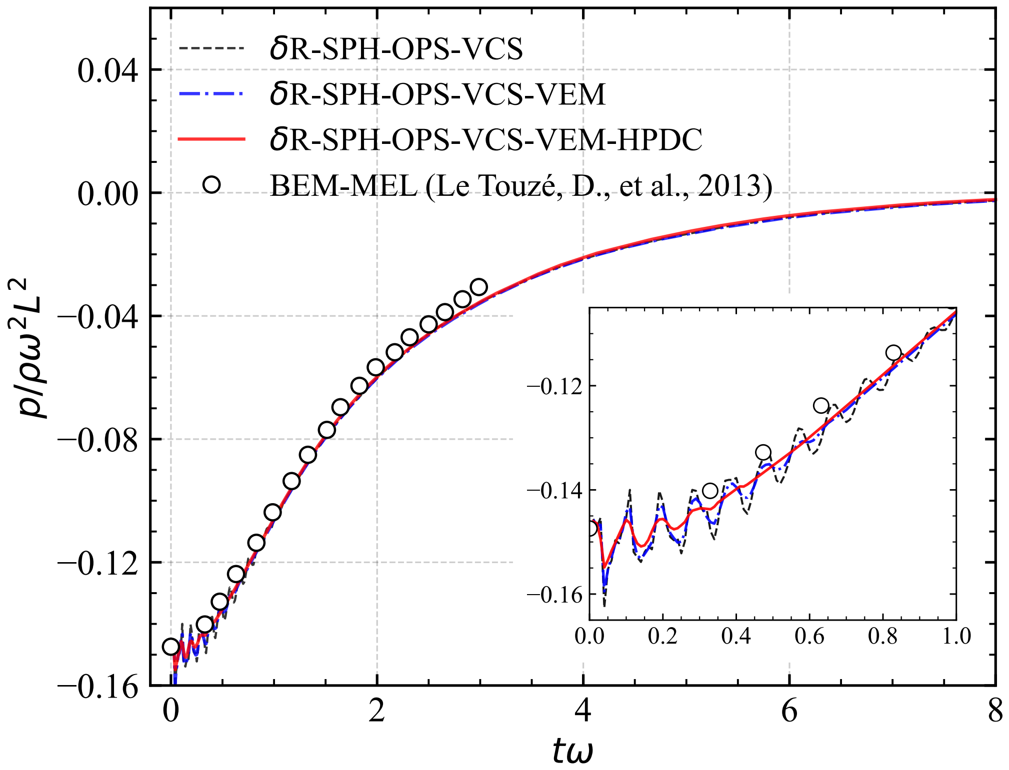
- Unphysical pressure fluctuations are mitigated by VEM and HPDC

# 3.1 A rotating fluid square: Kernel summation density field



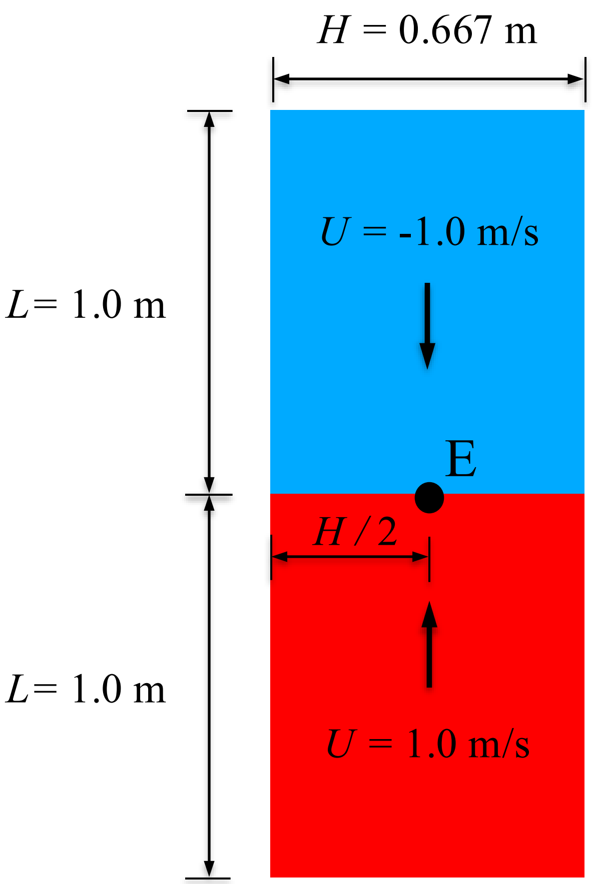

- VCS is effective in maintaining volume conservation

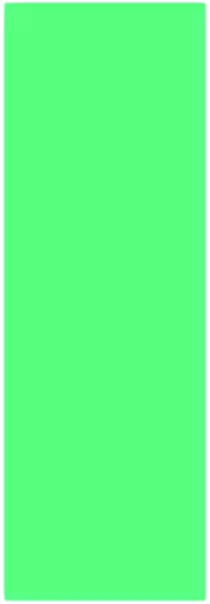
➢ Pressure histories at the square center




- Unphysical pressure fluctuations are mitigated by VEM and HPDC

# 3.2 Impact of two fluid patches: Numerical settings



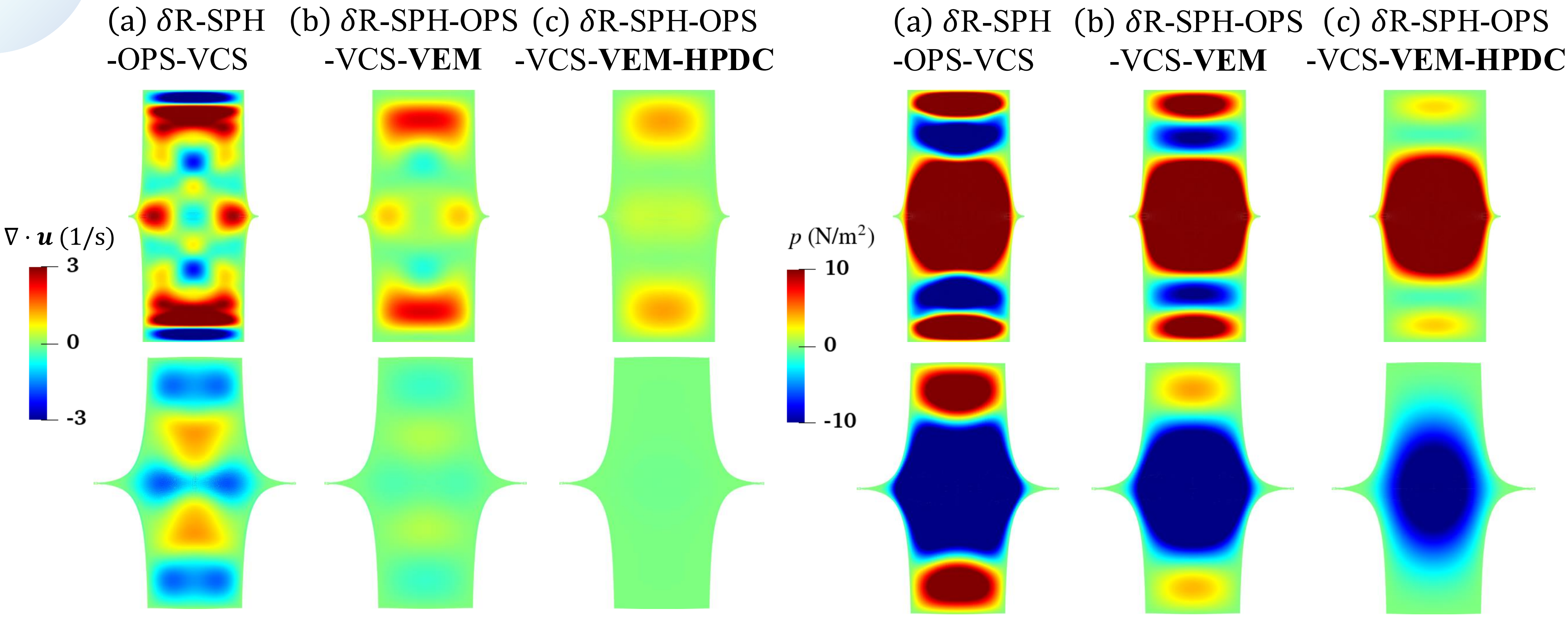
$L$ = 1 m, $dp$ = 0.005 m

$C_0$ = 100.0 m/s

Predictor-Corrector, $C_{CFL}$ = 0.3

Kernel = $5^{th}$ Wendland C2, $h/dp$ = 2

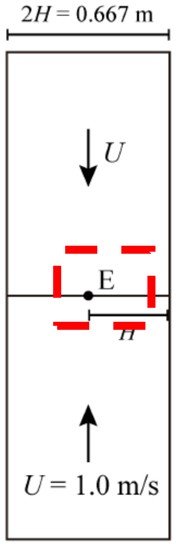
# 3.2 Impact of two fluid patches: Velocity divergence and pressure

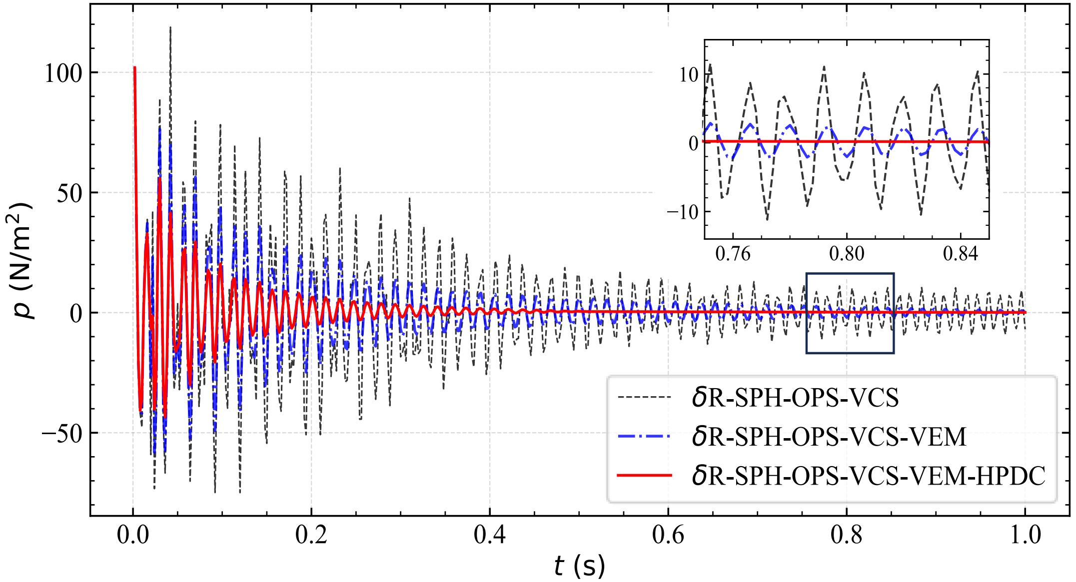



- Progressively adopting VEM and HPDC, velocity divergence errors reduced to nearly zero
- Compression/rarefaction waves partially reduced by VEM and further mitigated by the combination of VEM and HPDC

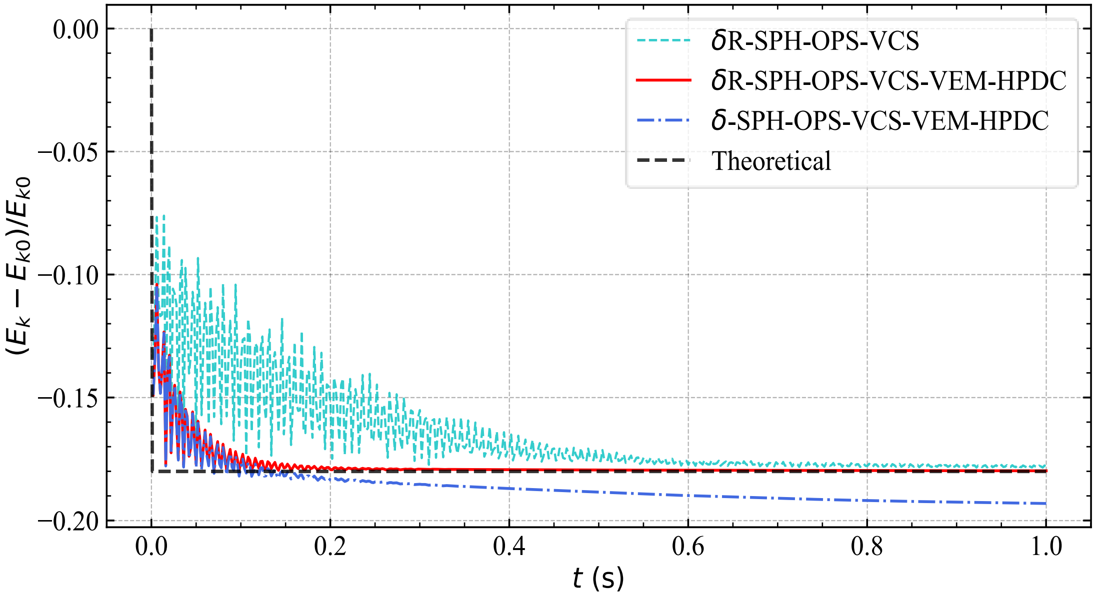
# 3.2 Impact of two fluid patches: Pressure and energy

➢ Pressure histories at measuring point




➢ Normalized mechanical energy




■ Combination of VEM and HPDC

↓

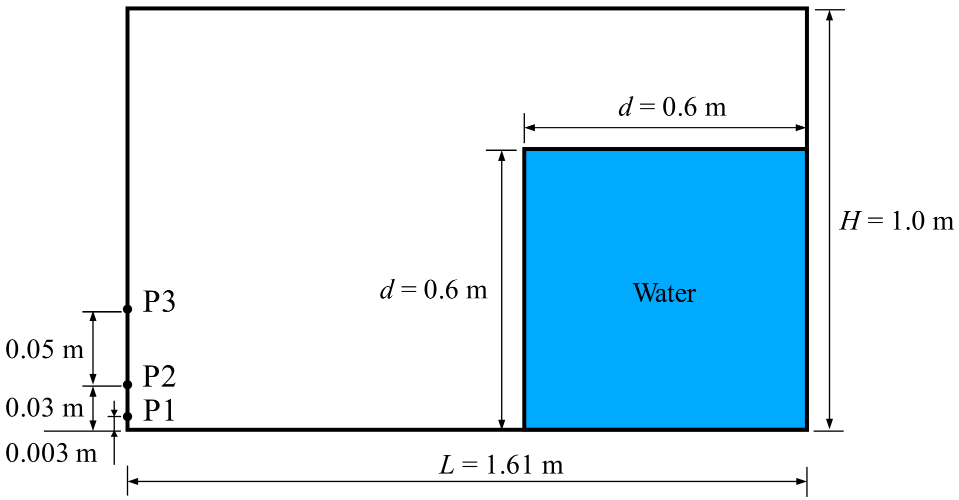
Reduces the magnitude of fluctuations by 88.9%

■ VEM and HPDC → Effectively eliminate the acoustic waves and the associated energy

■ Prediction by δR-SPH-OPS-VCS-VEM-HPDC model converges to theoretical solution

# 3.3 2D dam break: Pressure fields

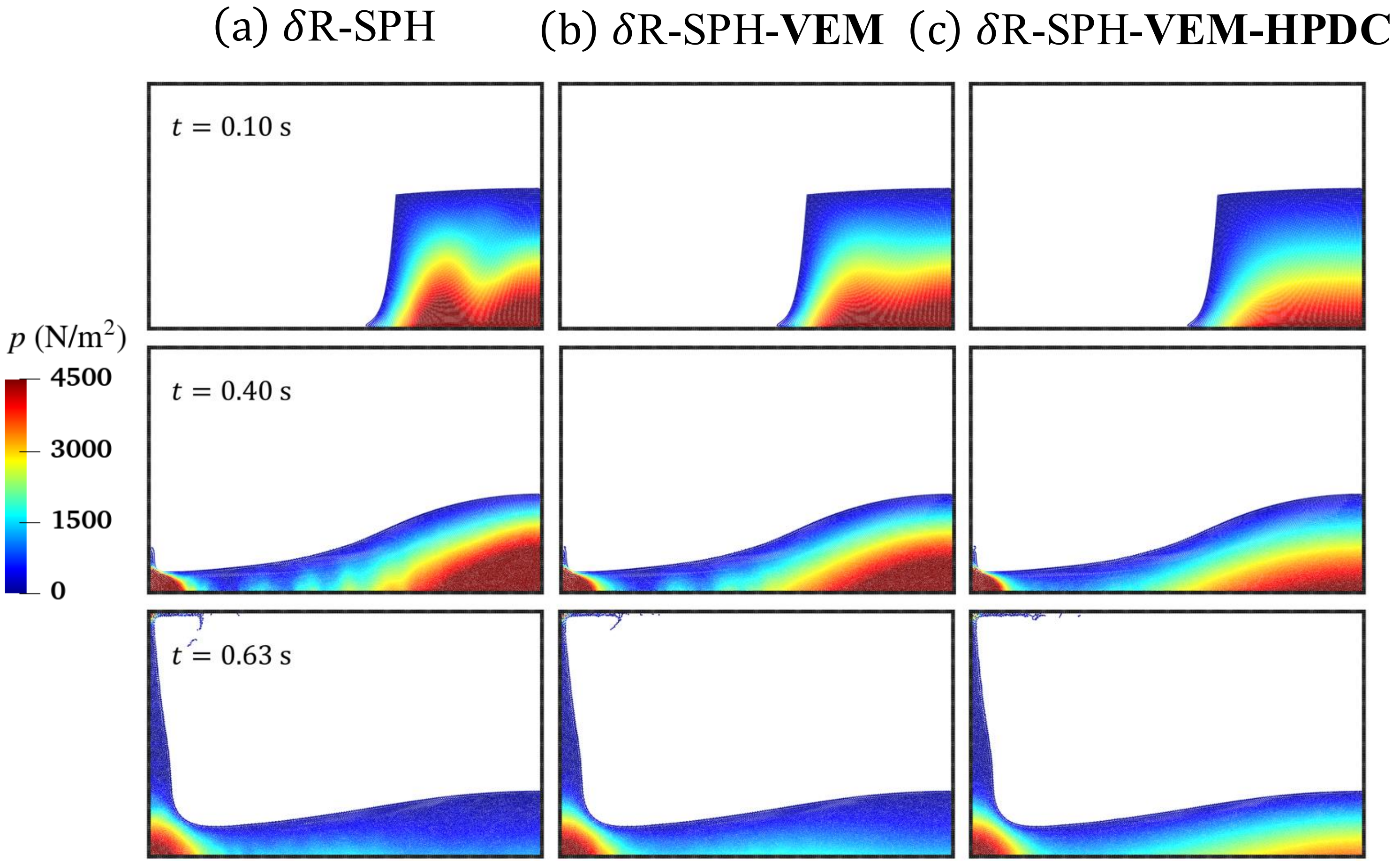
➢ Numerical settings




$dp$ = 0.005 m  $C_0 = 10 * \sqrt{2gd}$ m/s

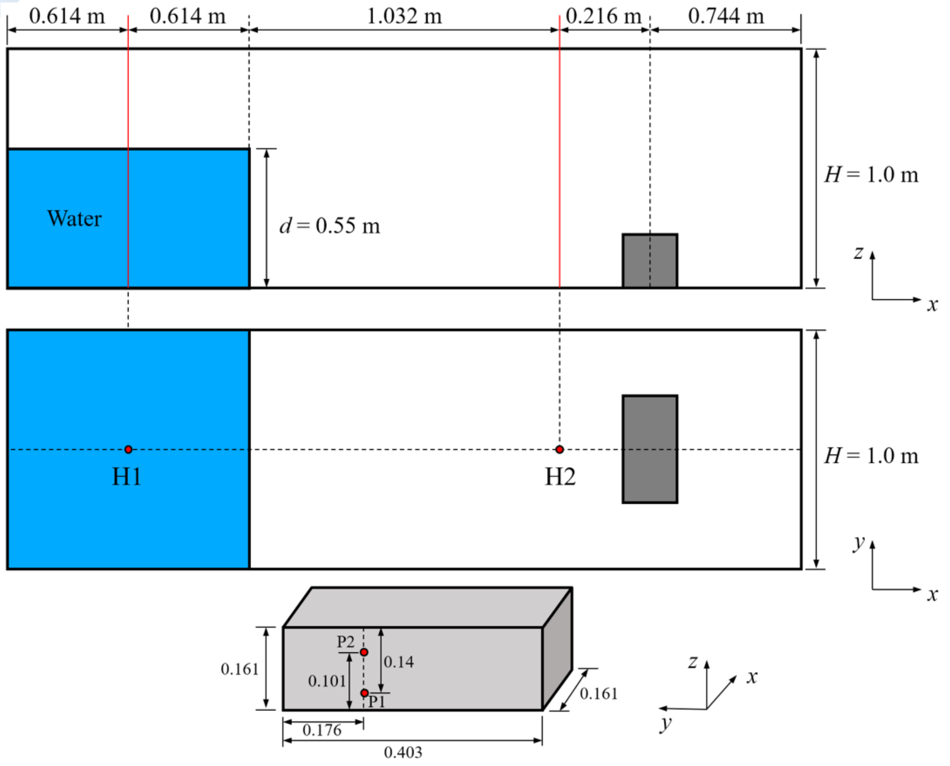
Predictor-Corrector, $C_{CFL}$ = 0.2

Kernel = 5th Wendland C2, $h/dp$ = 2

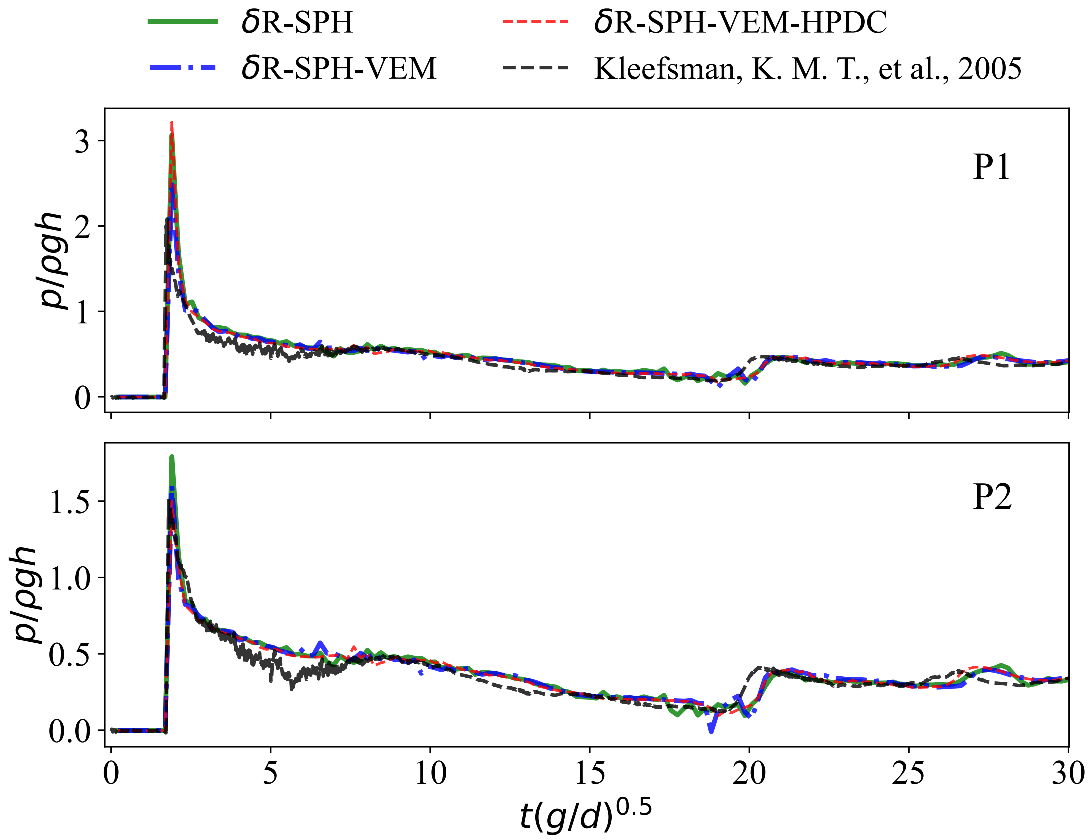



■ Pressure noises partially removed by the VEM and almost eliminated by further incorporation of HPDC

# 3.4 3D dam break: Pressure histories

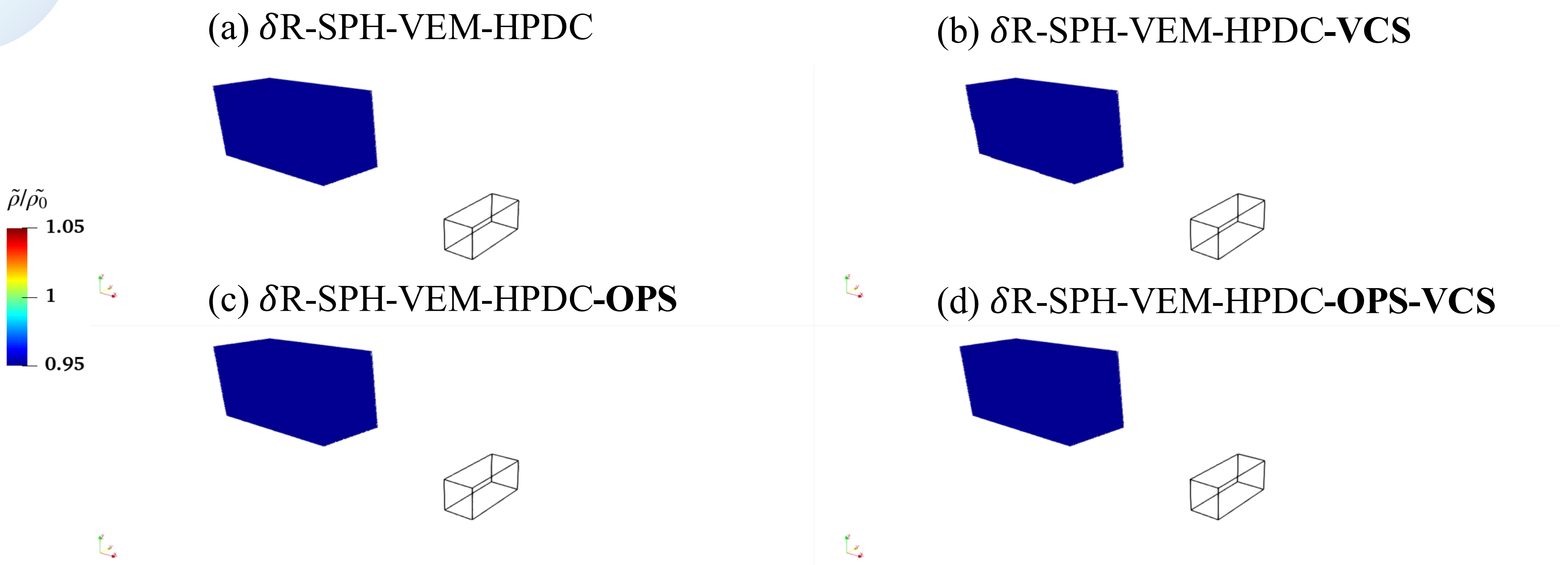



$dp$ = 0.01 m, $N \approx$ 1200000, $C_0 = 10 * \sqrt{2gd}$ m/s



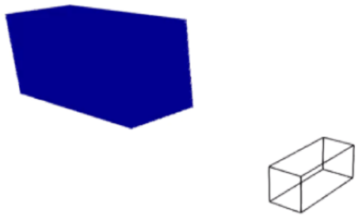

- All schemes are implemented on both CPU and GPU → Efficient 3D simulations
- Pressure results are in satisfactory agreement with the experimental data

# 3.4 3D dam break: Kernel summation density field




- Global and local volume non-conservation is observed in the δR-SPH-VEM-HPDC results
- OPS → Exacerbates volume expansion but improves particle distribution
- VCS → Maintain density invariance and volume conservation effectively

# 3.4 3D dam break: Computational efficiency and memory usage



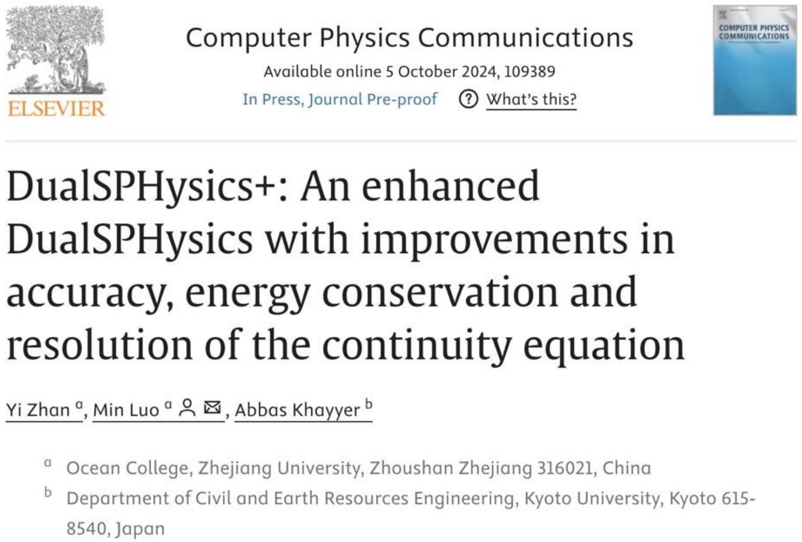
1,160,663 particles, 1 s physical time

| | Total steps | GPU time (s) | GPU time per step (s) | GPU time per physical second (s) | GPU Memory usage (MB) |
|---|---|---|---|---|---|
| $\delta$-SPH | 14914 | 629.7 | 0.042 | 629.7 | 340.9 |
| $\delta$R-SPH | 14825 | 657.3 | 0.044 | 657.3 | 354.2 |
| $\delta$R-SPH-OPS | 14649 | 924.9 | 0.063 | 924.9 | 371.9 |
| $\delta$R-SPH-OPS-VCS | 14709 | 1030.0 | 0.070 | 1030.0 | 400.4 |
| $\delta$R-SPH-OPS-VCS-VEM | 14425 | 1075.3 | 0.075 | 1075.3 | 418.1 |
| $\delta$R-SPH-OPS-VCS-VEM-HPDC | 14443 | 1098.5 | 0.076 | 1098.5 | 431.4 |

- $\delta$R, OPS, VCS, VEM and HPDC increase the computational time by 4.4%, 42.5%, 16.7%, 7.2%, 3.7% and the memory usage by 3.9%, 5.2%, 8.3%, 5.2% and 3.9%

# 4 Summary: DualSPHyscis+

- Combination of VEM and HPDC → Cleans the velocity divergence errors effectively
- $\delta$R-SPH → Retains numerical stability and mitigates excessive energy dissipation
- OPS → Implements particle shifting at and in the vicinity of free surfaces properly
- VEM, HPDC and VCS → Improve enforcement of velocity divergence-free and density invariance conditions, enhancing the resolution of the continuity equation.
- On going/ future work → Fluid-structure interactions, reinforcement learning, other engineering applications


ELSEVIER

Computer Physics Communications

Available online 5 October 2024, 109389

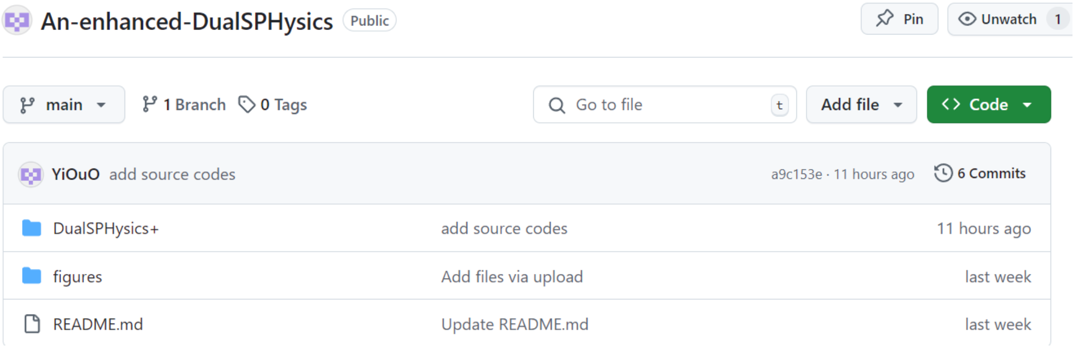
In Press, Journal Pre-proof ⓘ What's this?

DualSPHysics+: An enhanced DualSPHysics with improvements in accuracy, energy conservation and resolution of the continuity equation

Yi Zhan [a], Min Luo [a], Abbas Khayyer [b]

[a] Ocean College, Zhejiang University, Zhoushan Zhejiang 316021, China

[b] Department of Civil and Earth Resources Engineering, Kyoto University, Kyoto 615-8540, Japan




https://github.com/YiOuO/An-enhanced-DualSPHysics

**Parallelization on both the CPU (OpenMP) and GPU (CUDA)**

# Thank you for your attention